EFFECTIVE NUCLEAR POTENTIALS IN s-d SHELL

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ABSTRACT. The radial integrals, in terms of which the effective interaction in nuclear shell model has been parameterized, have been calculated for various potentials and these are compared with the values obtained by Cohen, Lawson and Pandya. The potentials that have been tried include some free nucleon-nucleon potentials proposed earlier and which fit the scattering data well.

INTRODUCTION

The spectra of the oxygen isotopes have been analysed in the frame-work of the nuclear shell model to derive information about the effective nucleon-nucleon interactions in s-d shell. In the recent calculations of Cohen, Lawson and Pandya (1967) the interaction in $T = 1$ states is parameterised in terms of several radial integrals of the effective potential. These radial integrals I_{nl} are defined as follows

$$I_{nl} = \int_0^\infty R_{nl}^2(r) V_{nl}(r) r^2 dr$$

where $R_{nl}(r)$ is the harmonic oscillator radial wavefunction, $\vec{r} = \vec{r}_2 - \vec{r}_1$ is the relative coordinate of the two particles, and V_{nl} is the effective potential, assumed to be central, but possibly l-dependent. In this representation, the triplet odd part of the potential is described by only two parameters, viz, $I_{0p}+I_{0f}$ and $I_{0p}+I_{1p}$. In view of the fact that such a representation is only a very crude caricature of the realistic nucleon-nucleon force in triplet odd state, we do not consider a simple potential representation for this part. In this paper we shall only consider the singlet even component of the effective interaction.

Cohen, Lawson and Pandya have deduced the values of the parameters I_{nl} from a least-squares-fit analysis to the observed spectra of oxygen isotopes. The values of the five parameters representing the singlet even interaction are given in table 1 under the column CLP. The five parameters are

$$I_{0s}+I_{0g},\ I_{0s}+I_{1d},\ I_{0s}+I_{2s},\ I_{1s},\ I_{0d}.$$

The simple problem we consider here is to see if one can find a potential which will give rise to the CLP values for the radial integrals, and the nature of such a potential. We consider only Gaussian shapes for the potentials (including sums of Gaussians), but the results for Yukawa shapes are expected to be quite similar. We have then in general

$$V = \sum_i V_i \exp(-\mu_i r^2).$$

Since the harmonic oscillator wavefunction has the factor $\exp\left(-\frac{\nu}{2}r^2\right)$, the radial integrals can be shown to depend only on the dimensionless parameters $\lambda_i = (\nu/\mu_i)^{\frac{1}{2}}$. Table 2 gives the expressions for the radial integrals in units of V and in terms of $x = \lambda^2/(1+\lambda^2)$. For all the calculations, we take $\nu = 0.346\ \text{fm}^{-2}$ (Dawson *et al*, 1962).

RESULLTS AND DISCUSSION

First of all we quote the results for a single Gaussian potential, for which we take $\lambda = 0.6$, 0.9 and 1.2 and fix V_0 to give the correct value for $I_{0s}+I_{0g}$. The results are shown in columns 3, 4, 5 of table 1. We note that for the short range potential ($\lambda = 0.6$) the value of I_{0d} is reasonably small, but the values of $I_{0s}+I_{2s}$ and I_{1s} remain large compared to the CLP values. On the other hand as the range is increased, $I_{0s}+I_{2s}$ comes closer to the CLP values, but I_{0d} and I_{1s} remain too large.

It is then interesting to inquire if the result can be improved by considering a sum of Gaussians. It is now well known that a realistic potential contains a repulsive core, presumably a soft one, and some of this repulsive effect may carry over into an effective nuclear force. We therefore assume for the effective potential a sum of three Gaussian terms of suitably fixed ranges (i.e. λ_i fixed) and try to make a least-squares-fit to the five radial integrals by varying the V_i. The fixed values of the ranges and the values of V_i that give the best fit to radial integrals CLP are shown in table 3. The values of the radial integrals obtained for these three cases are also shown in table 1. There appears to be hardly any substantial improvement in the values of the integrals, in particular the values of I_{0d} and I_{1s} are still quite large compared to the CLP values. This may partly be due to the fact that we have fixed the longest range for the potentials to have about the one-pion-exchange-potential value. The value of I_{0d} seems to demand a shorter range than a one-pion-exchange-potential would give.

An amazing feature of the empirical potential form derived here is that the shortest-ranged term in it turns out to be attractive in all cases. A plot of the potentials I and III is shown in fig. 1. The potential beyond about 1 fm is weak and attractive, becomes slightly repulsive or zero in the intermediate range, $0.5 < r < 1.0$ fm, but in the inner region $r < 0.5$ fm the potential becomes attractive, rather strongly so. To understand this, we consider the Scott-Moszkowski (1960) prescription for obtaining an effective interaction from a realistic potential which contains a strong repulsive core. The Scott-Moszkowski method in its

Table 1

Values of radial integrals defining singlet even potential (in MeV)

	CLP	Simple Gaussian $\lambda=0.6$ $V=-64.89$	0.9 -28.49	1.2 -17.64	Least-Squares-Fit to 3 Gaussians I	II	III	ITTY (1963)			EH(1966)	HV(1966)
$I_{0s}+I_{0g}$	− 8.89	− 8.89	− 8.89	− 8.89	− 7.20	− 7.45	− 7.14	− 8.57	− 2.57	+ 7.83	− 7.60	−13.50
$I_{0s}+I_{1d}$	− 8.01	−10.06	−10.71	−10.62	− 9.13	− 8.94	− 8.84	−12.22	− 6.42	+ 3.64	−11.30	−16.90
$I_{0s}+I_{2s}$	−13.33	−15.25	−12.94	−11.75	−12.36	−12.32	−12.83	+ 1.30	+20.10	+40.90	+ 0.60	−18.20
I_{1s}	− 2.52	− 7.79	− 5.61	− 4.82	− 4.77	− 4.80	− 4.26	+ 1.60	+11.30	+16.30	+ 2.40	− 6.90
I_{0d}	+ 0.07	− 0.65	− 1.71	− 2.79	− 1.81	− 2.20	− 2.09	− 2.46	− 2.54	− 2.57	− 1.70	− 3.10

Table 2

Radial Integrals in units of V_0 : $x = \lambda^2/(1+\lambda^2)$.

$I_{0s} = x^{3/2}$

$I_{1s} = (1/2)x^{3/2}(3-6x+5x^2)$

$I_{2s} = (1/8)x^{3/2}(15-60x+130x^2-140x^3+63x^4)$

$I_{0d} = x^{7/2}$

$I_{1d} = (1/2)x^{7/2}(7-14x+9x^2)$

$I_{0g} = x^{11/2}$

simplest version amounts to removing the hard-core plus a part of the attractive potential upto some distance $r(\sim 1 \text{ fm})$ such that the repulsive core and the attractive potential being removed balance out, in the sense that together they would

Table 3

Parameters of Three-Gaussian Models fitted to CLP values

	λ_1	λ_2	λ_3	V_1 (MeV)	V_2 (MeV)	V_3 (MeV)
I	0.176	0.470	0.883	−743.93	98.88	−37.04
II	0.235	0.529	0.883	−505.60	125.80	−47.74
III	0.294	0.588	0.883	−202.96	101.30	−52.56

give a zero phase shift for *s*-wave nucleon-nucleon scattering. The effective potential between nucleons in nuclei is then simply the remaining weak tail of the attractive potential. Our three-Gaussian model for the effective potential has roughly this weak, attractive character in the region $r > 0.8-1.0$ fm. Where does the strong, short-range attraction then come in ? It is known (Dawson *et al*, 1962; Kuo *et al*, 1966), that if the spectrum of O^{18} is computed using an effective potential (or reaction matrix) obtained from some suitable hard-core realistic potential, the binding of the ground state 0^+ is predicted to be much too weak by about an MeV. Kuo *et al* showed that this defect can be corrected for by renormalising the two-body reaction-matrix-elements taking into account the excitation of nucleons from the O^{16} core. The core-excitation effect on the singlet-even part of the effective potential appears to be equivalent to the addition of a short-ranged attractive potential—in some sense a pairing force. The three-Gaussian model presented here seems to suggest that the effective singlet even potential consists of a weak attractive part in the region $r > 0.8$ fm. (as required by the Scott-Moszkowski method of constructing a reaction matrix) plus a short-ranged attractive component to take account of the core excitation effects on two-body matrix elements.

Finally, we briefly consider several three-Gaussian potential models proposed by various authors. Ishihara *et al* (1963) have considered models which have one-pion exchange-potential tail, and have repulsive soft core. The range and strength parameters fit the singlet scattering length, effective range and phase-shifts upto 310 MeV. Some of the parameters used by them are shown in table 4. The results for the radial integrals with these potentials are also shown in table 1.

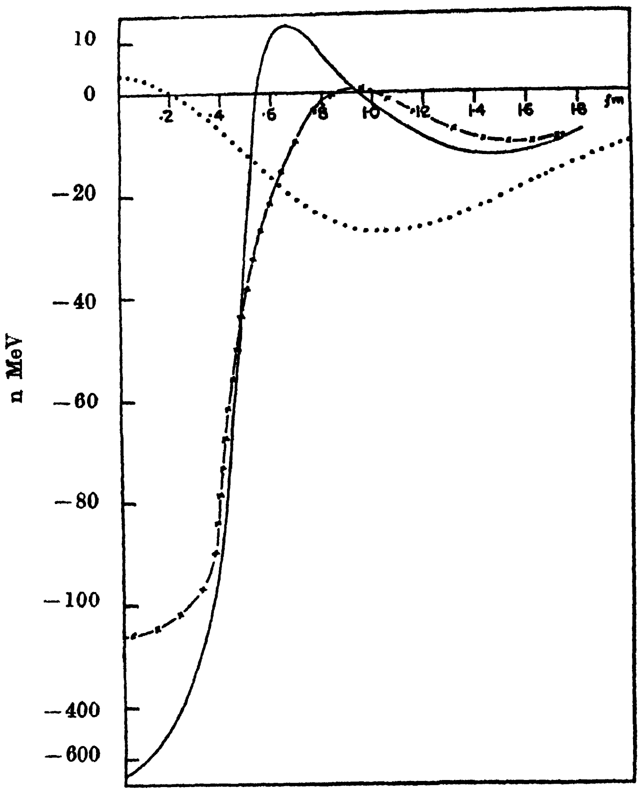

Fig. 1. Radial dependence of Potentials

———————— Three—Gaussian-model potential I

—×—×— Three—Gaussian-model potential III

.... Hughes and Volkov (1966)

A similar potential has also been proposed by Eikomeier and Hackenbroich (1966), the parameters (ranges and depths) again being fitted to the binding energy of the deuteron as well as Yale (*YLAM*) phase shifts upto 300 MeV laboratory energy. Their parameters for the singlet even potential are shown in table 4. The potential is again strongly repulsive in the region $r < 0.7$ fm. The radial integrals calculated with this potential are also shown in table 1. We note that for this case as well as for the Ishihara (1963) models the repulsive core, although soft, is so strong that the radial integrals I_{1s} and I_{2s} become positive ! This is quite natural since for application to nuclear spectroscopy, even the soft repulsive core has to be removed by introducing a reaction-matrix.

On the other hand Hughes and Volkov (1966) have proposed a simple two-Gaussian model, the parameters of which are fitted to nuclear properties such as correct equilibrium size and binding energy of He^4, and the spectra of $1p$ shell nuclei. The parameters of this model, listed in table 4, give values of radial

Table 4

Parameters of Potentials	
Ishihara *et al* (1963) Potential : $V(r) = \sum_{i=1}^{3} V_i \exp(-\mu_i r^2)$	
$V_1 = 7.2$ MeV :	$\mu_1 = 0.284$ fm^{-2}
$V_2 = -279$ MeV	$\mu_2 = 1.125$ fm^{-2}
V_3 in MeV	$\mu_3 =$ in fm^{-2}
1000	3.521
2000	5.005
4000	6.757
Eikemeier *et al* (1966) Potential : $V(r) = \sum_{i=1}^{3} V_i \exp(-\mu_i r^{-2})$	
$V_1 = 880$ MeV	$\mu_1 = 5.40$ fm^2
$V_2 = -70$,,	$\mu_2 = 0.64$,,
$V_3 = -21$,,	$\mu_3 = 0.48$,,
Hughes *et al* (1966) Potential : $V(r) = -V_a \exp(-\mu_a r^2) + V_r \exp(-\mu_r r^2)$	
$V_a = 58.52$ MeV	$\mu_a = 0.444$ fm^{-2}
$V_r = 62.10$,,	$\mu_r = 1.776$,,

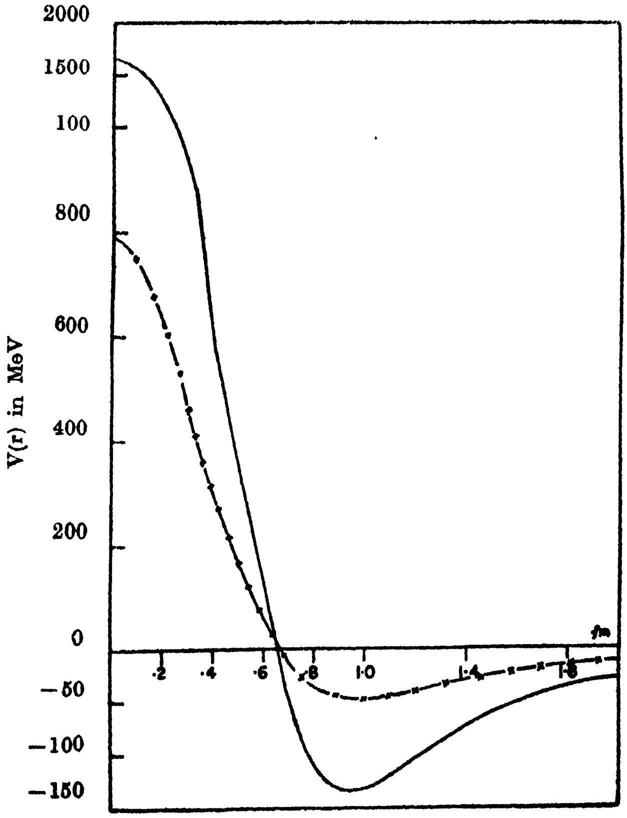

Fig. 2. Radial dependence of potentials

——————— Ishihara *et al* (1963)

—×—×— Eikemeier and Hackenbroich (1966)

integrals listed in table 1. We now note that this potential inspite of having a repulsive core gives approximately correct values for the radial integrals. In fact, it appears to be more attractive than our three-Gaussian model which contains an attractive core!

The potentials discussed above are plotted in figures 1 and 2.

SUMMARY

We have tried to determine the parameters of a potential (taken as a sum of three Gaussians for sufficient flexibility) in singlet even states of two nucleons so as to give the best fit to the values of the radial integral parameters of Cohen, Lawson and Pandya (1967) which give a good fit to the level schemes of oxygen isotopes. The result shows a weak attractive potential in the region $r > 0.8$ fm, but a strong attractive short-range part as well. This latter feature is interpreted as the result of core-excitation effects of Kuo and Brown (1966).

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